Action Models for Coalition Logic

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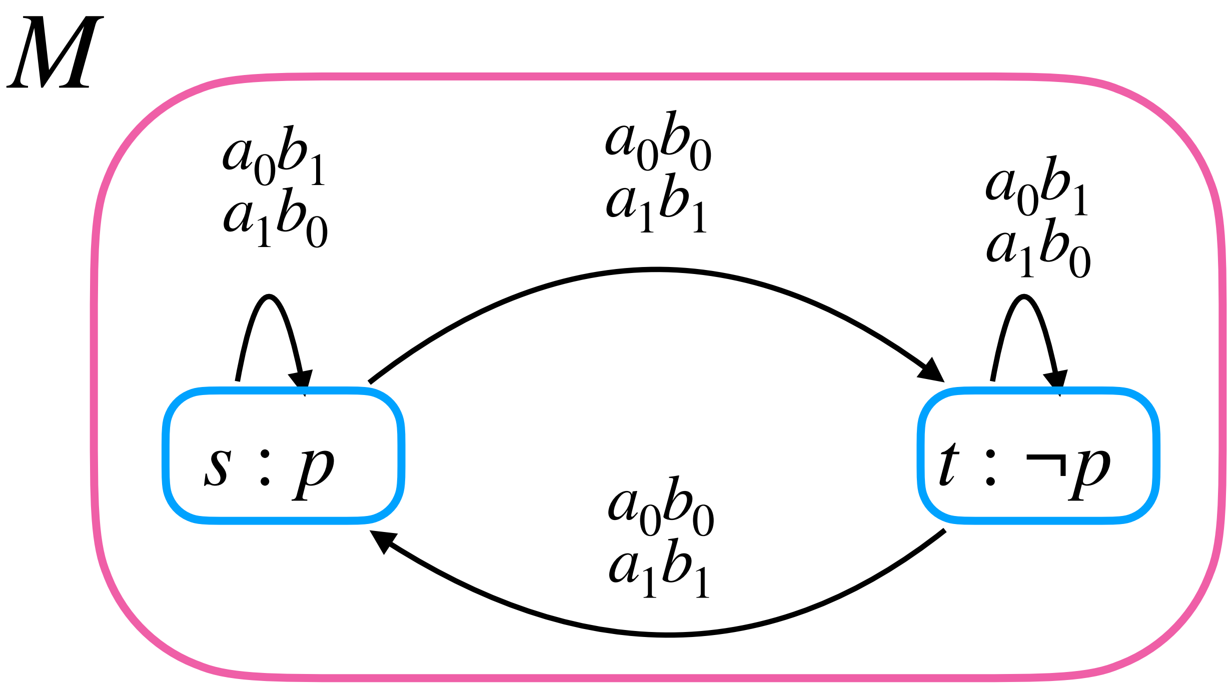
Coalition logic

- Coalition logic (CL) [Pauly, 2002] is used to reason about abilities of groups of agents in the presence of opponents
- Language of CL: $\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle\!\langle C \rangle\!\rangle \varphi$
- $\langle\!\langle C \rangle\!\rangle \varphi$ is read as 'coalition C can bring about φ by a joint action no matter what agents outside of the coalition do.'
- Dual $[\![C]\!]\varphi$ is read as 'coalition C cannot avoid φ'

Example



p: agents receive a *p*rize

$M_t \models \neg p \wedge \langle\langle \{a, b\} \rangle\rangle p$

$M_t \models [\![a]\!] \neg p \wedge [\![b]\!] \neg p$

Models

A concurrent game model (CGM) M is a tuple (S, Act, act, out, L), where

- S is a non-empty set of states;
- Act is a non-empty set of actions;
- act assigns to each agent and each state a non-empty set of actions;
- out assigns to each state and each combination of actions available to agents a unique outcome state;
- L is the valuation function.

We will denote by α_C a set of actions such that for each $i \in C$ there is exactly one action of i in α_C.

Semantics

The semantics of $\langle\langle C \rangle\rangle \varphi$ is

$$M_s \vDash \langle\langle C \rangle\rangle \varphi \text{ iff } \exists \alpha_C, \forall \alpha_{\overline{C}} : M_t \vDash \varphi, \text{ where } t = out(s, \alpha_C \cup \alpha_{\overline{C}})$$

The semantics of $[\![C]\!]\varphi$ is

$$M_s \vDash [\![C]\!]\varphi \text{ iff } \forall \alpha_C, \exists \alpha_{\overline{C}} : M_t \vDash \varphi, \text{ where } t = out(s, \alpha_C \cup \alpha_{\overline{C}})$$

Dynamic coalition logic

- We can consider a CGM as a protocol or a contract that specifies what agents can and cannot do in different situations
- CL, however, cannot capture updates of such a policy
- Thus, we propose a study of dynamic coalition logic

$$\boxed{M_1} \xrightarrow{\text{Update 1}} \boxed{M_2} \xrightarrow{\text{Update 2}} \boxed{M_3} \xrightarrow{\text{Update 3}} \boxed{M_4}$$

Inspired by the dynamic epistemic logic (DEL)

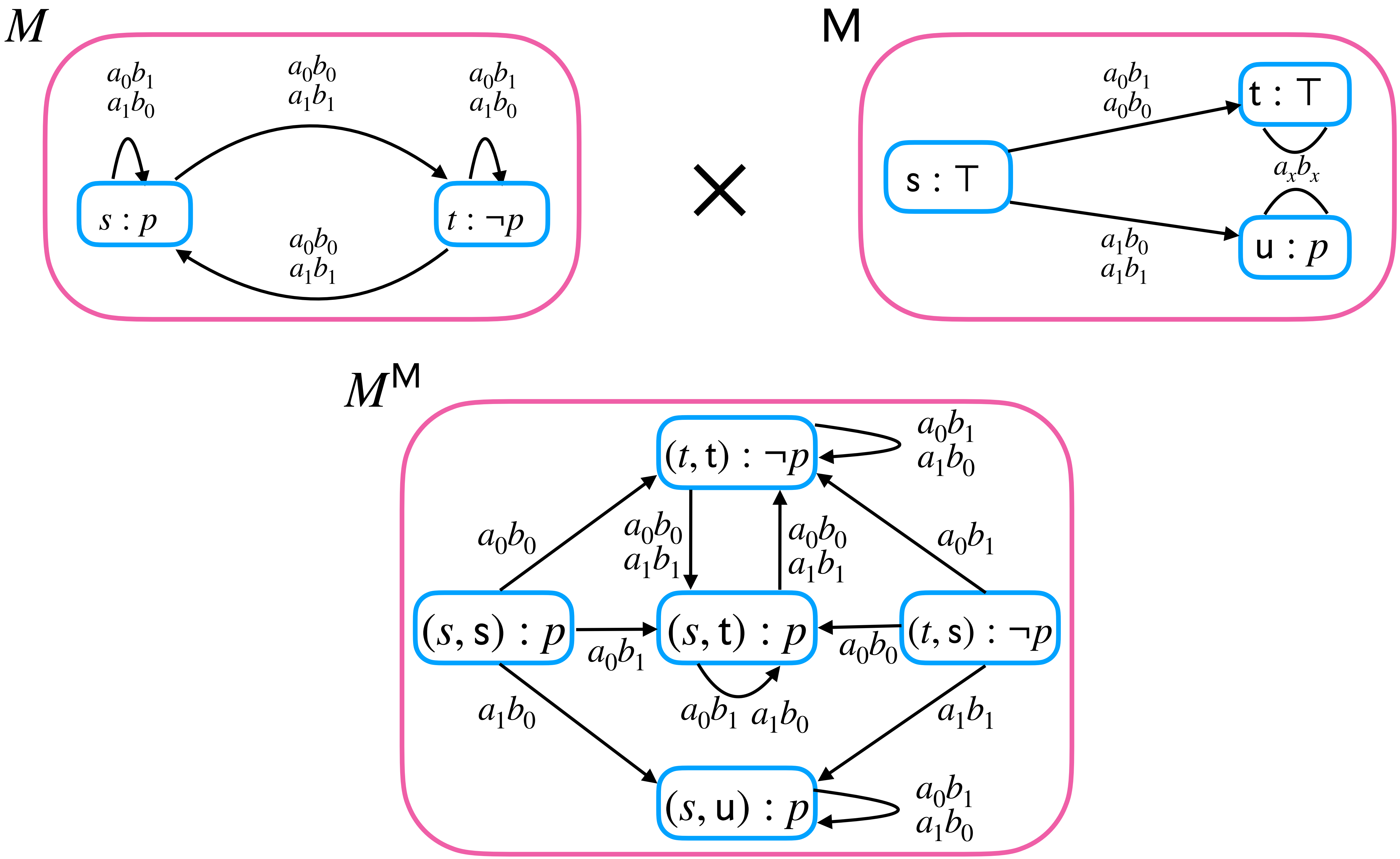
Action models

- To capture updates, we borrow the idea of model updates from action model logic (AML) [Baltag, Moss 2004]

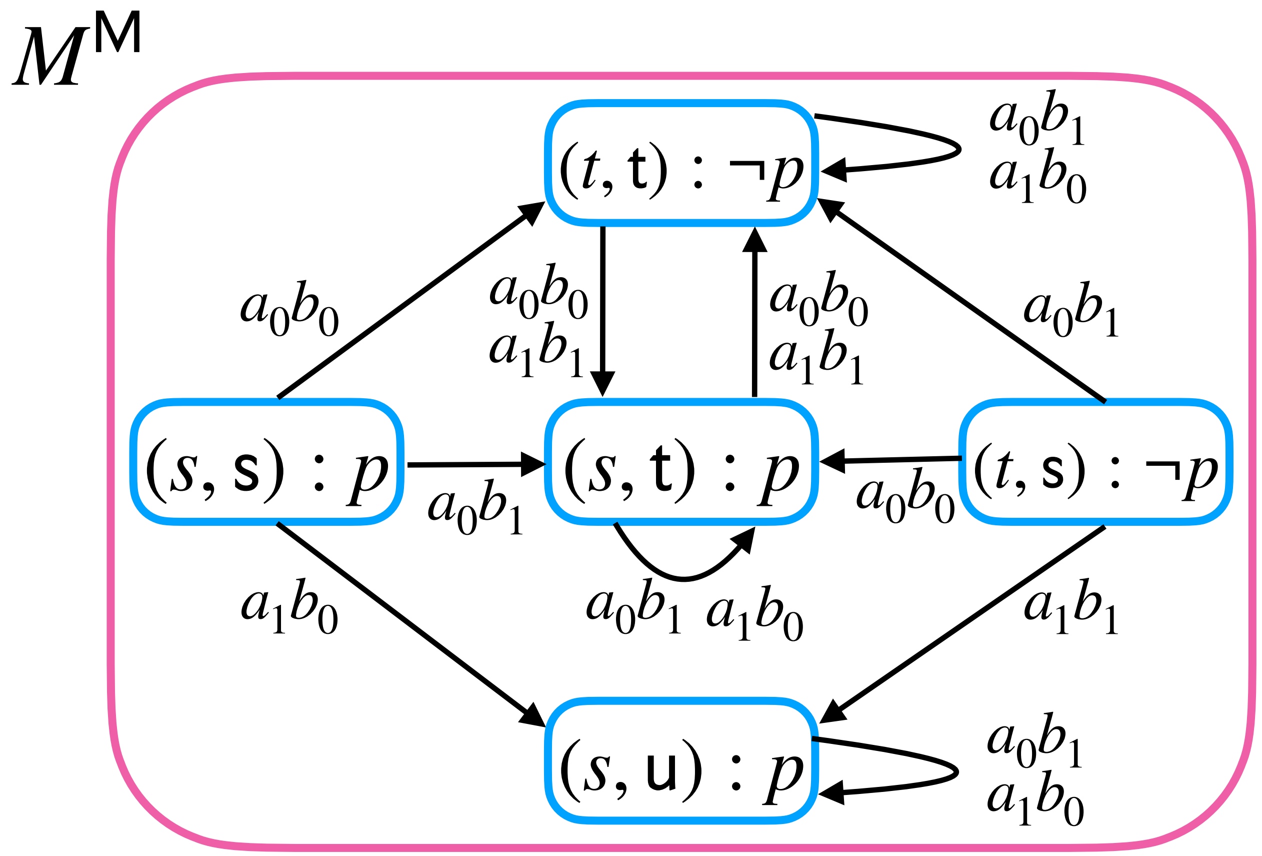
- The language of Coalition Action Model Logic (CAML) is
  $\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle\langle C \rangle\rangle\varphi \mid [\pi]\varphi$
  $\pi ::= \mathsf{M}_s \mid (\pi \cup \pi)$

- M_s is an action model, and $\pi \cup \rho$ is non-deterministic choice

Update example



A little problem

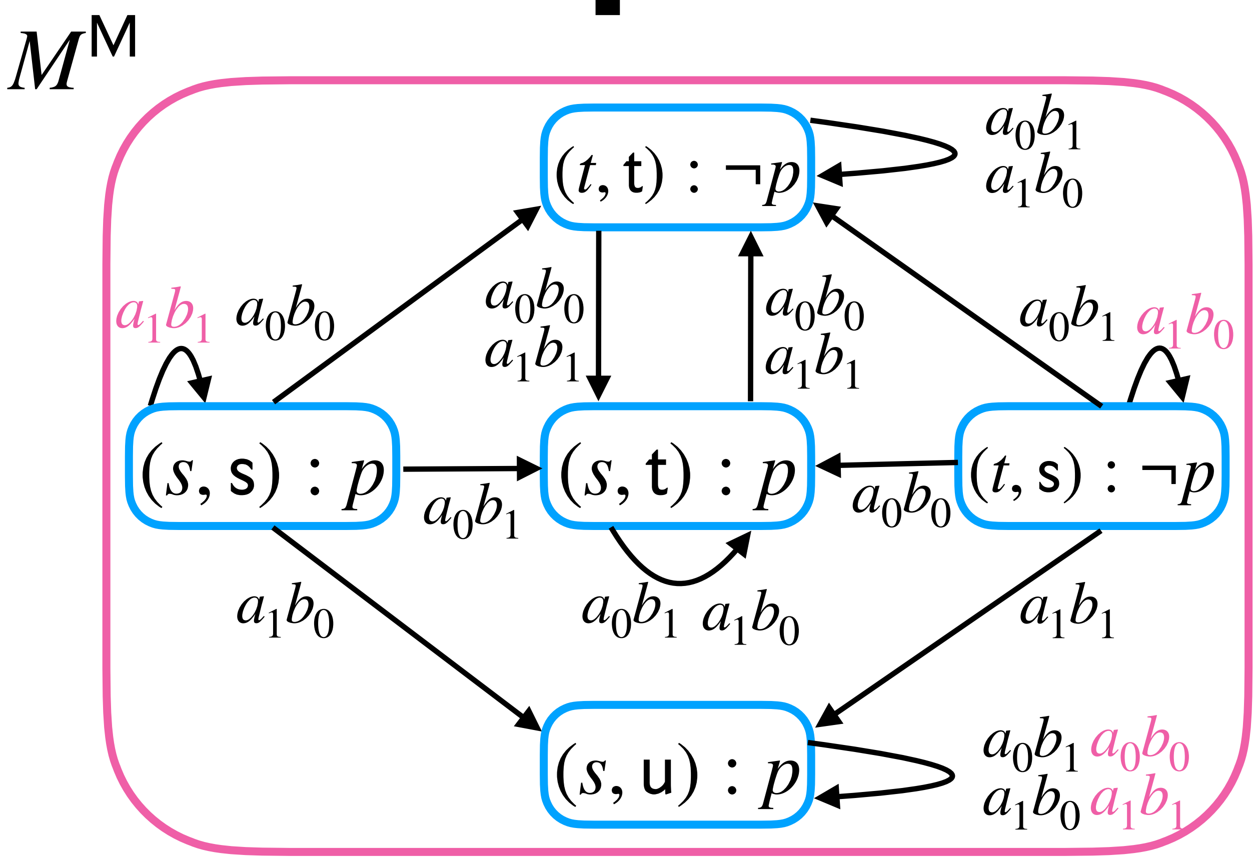


Transitions from some of the states are not defined for all action profiles

In other words, M^{M} is not a model!

This is the result of the fact that (some parts of) a new modification contradicts the existing protocol

A little problem

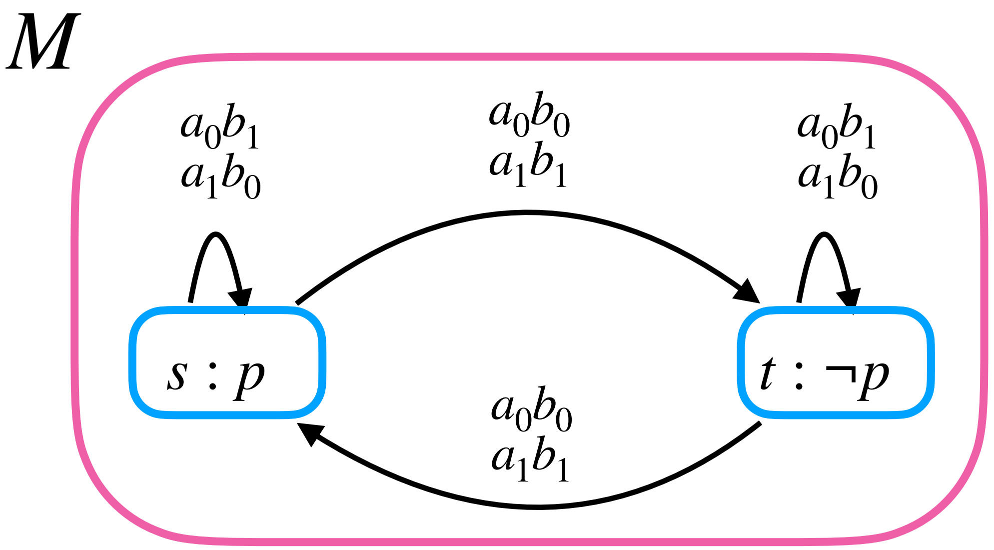


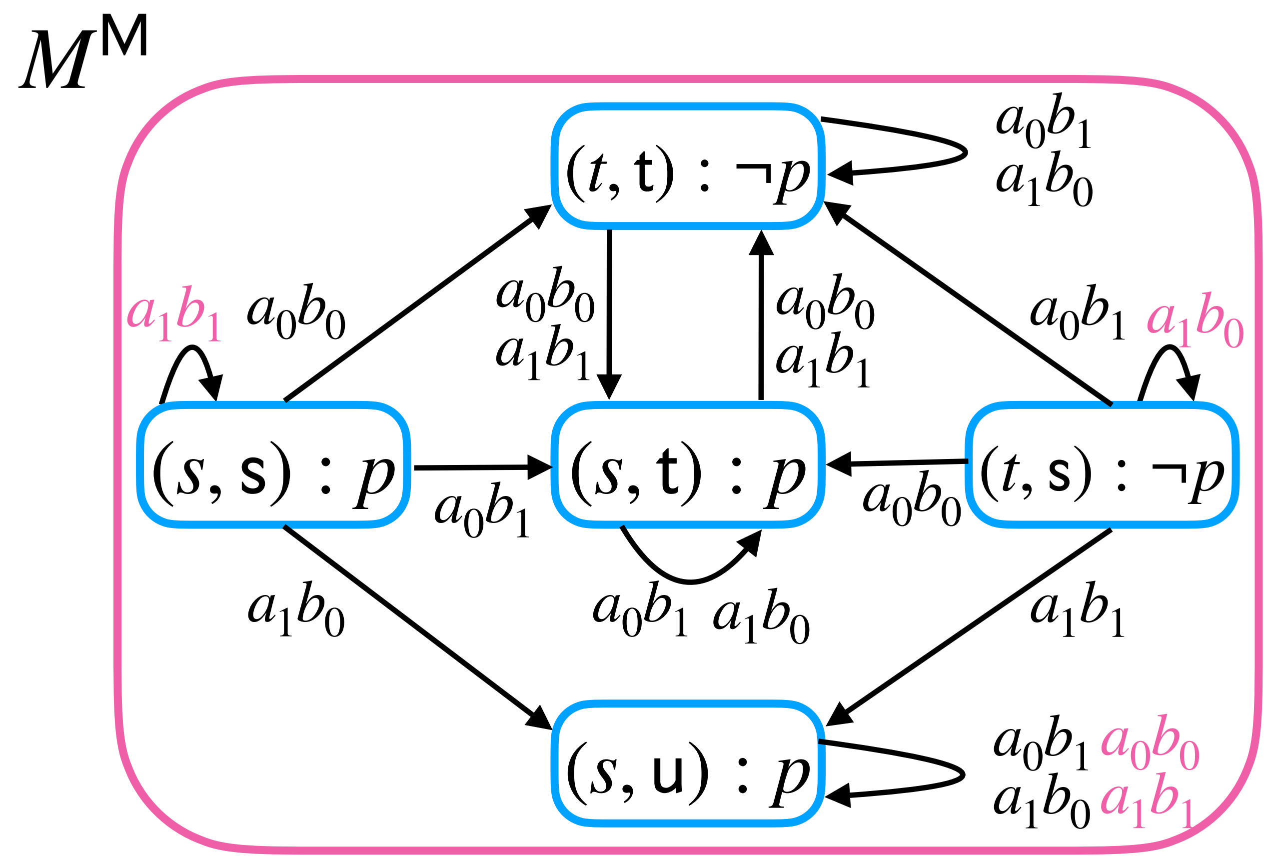
Transitions from some of the states are not defined for all action profiles

In other words, M^{M} is not a model!

Conservative approach: when in doubt, remain where you are

A little problem





Action model updates strategic abilities based on what agents can actually achieve in a given CGM

$$M_s \not\models \langle\!\langle \{a, b\} \rangle\!\rangle [\![\{a, b\}]\!] p$$

$$M_s \models [\mathsf{M}_\mathsf{s}] \langle\!\langle \{a, b\} \rangle\!\rangle [\![\{a, b\}]\!] p$$

Action models

An action model M is a tuple $(S, \mathsf{Act}, \mathsf{act}, \mathsf{out}, \mathsf{pre})$, where

- S is a non-empty finite set of states;
- Act is a non-empty set of actions;
- act assigns to each agent and each state a non-empty set of actions;
- out assigns to each state and some combinations of actions available to agents a unique outcome state;
- pre : $S \to \mathscr{CL}$ assigns to each state a formula of CL.

Semantics of CAML

Let M_s be a CGM and M_s be an action model.

$$M_s \vDash [\mathsf{M}_\mathsf{s}]\varphi \text{ iff } M_s \vDash \mathrm{pre(s)} \text{ implies } M^{\mathsf{M}}_{(s,\mathsf{s})} \vDash \varphi$$

$$M_s \vDash \langle \mathsf{M}_\mathsf{s} \rangle\varphi \text{ iff } M_s \vDash \mathrm{pre(s)} \text{ and } M^{\mathsf{M}}_{(s,\mathsf{s})} \vDash \varphi$$

$$M_s \vDash [\pi \cup \rho]\varphi \text{ iff } M_s \vDash [\pi]\varphi \text{ and } M_s \vDash [\rho]\varphi$$

$$M_s \vDash \langle \pi \cup \rho \rangle\varphi \text{ iff } M_s \vDash [\pi]\varphi \text{ or } M_s \vDash [\rho]\varphi$$

Proposition. The following schemata are valid:

1. $\langle \mathsf{M}_\mathsf{s} \rangle\varphi \rightarrow [\mathsf{M}_\mathsf{s}]\varphi$
2. $[\mathsf{M}_\mathsf{s}]p \leftrightarrow (\mathrm{pre(s)} \rightarrow p)$
3. $[\mathsf{M}_\mathsf{s}](\varphi \wedge \psi) \leftrightarrow ([\mathsf{M}_\mathsf{s}]\varphi \wedge [\mathsf{M}_\mathsf{s}]\psi)$
4. $[\pi \cup \rho]\varphi \leftrightarrow [\pi]\varphi \wedge [\rho]\varphi$

Similar to AML!

Expressivity

- Formulas φ and ψ are equivalent if for all M_s, $M_s \models \varphi$ iff $M_s \models \psi$

- Language $\mathscr{L}_1$ is at least as expressive as $\mathscr{L}_2$ ($\mathscr{L}_2 \leqslant \mathscr{L}_1$) if for all $\varphi \in \mathscr{L}_2$ there is an equivalent $\psi \in \mathscr{L}_1$. If $\mathscr{L}_2 \leqslant \mathscr{L}_1$ and $\mathscr{L}_1 \nleqslant \mathscr{L}_2$ we say that $\mathscr{L}_1$ is more expressive than $\mathscr{L}_2$. If $\mathscr{L}_2 \nleqslant \mathscr{L}_1$ and $\mathscr{L}_1 \nleqslant \mathscr{L}_2$, then $\mathscr{L}_1$ and $\mathscr{L}_2$ are incomparable

Theorem:

- CAML is more expressive than CL
- CAML and ATL are incomparable

Dissimilar to AML!

Corollary: formulas of CAML cannot be equivalently rewritten into formulas of CL

Model checking

Given a finite CGM M_s and a formula of CAML φ, the model checking problem consists in determining whether $M_s \models \varphi$

Theorem: the complexity of the model checking problem for CAML is *PSPACE*-complete

The result holds even for the case of a single agent

Recap and open questions

We proposed a study of dynamic coalition logic

We introduced action models for coalition logic in the vein of AML

We studied the relative expressivity of CAML and the complexity of the model checking problem

?Proof system for CAML?

?Variations of action models: ontic changes, granting new action to agents, revoking actions from agents?

?Extending the base language to ATL, ATL*, and SL?